लक्ष्यं विश्वमानम्

# WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 6th Semester Examination, 2023

## PHSADSE04T-PHYSICS (DSE3/4)

Time Allotted: 2 Hours | Full Marks: 50

*The figures in the margin indicate full marks.*
*Candidates should answer in their own words and adhere to the word limit as practicable.*
*All symbols are of usual significance.*

**Answer Question No.1 and any *two* questions from the rest**

1. Answer any ***fifteen*** questions from the following: $2\times15=30$

(a) Show that the identity element in a group is unique.

(b) Prove that the order of an element '$a$' of a group is same as that of its inverse i.e., '$a^{-1}$'.

(c) If '$a$' is conjugate to '$b$', '$b$' is conjugate to '$c$' then show that '$a$' is conjugate to '$c$'.

(d) If $A=\{1, 2, 3\}$, then write down the non-empty subsets of $A$.

(e) Show that the set $S$ of all integers does not form a group under the operation defined by

$$a*b=a-b \quad \forall\, a, b\in S.$$

(f) Let $f:N\to N: f(x)=2x$ for all $x$ in $N$. Check whether $f$ is a bijection when $N$ is the set of all positive integers.

(g) Show that the set $U(n)$ of all unitary matrices of order $n$, where $n$ is a fixed finite positive integer forms group under matrix multiplication.

(h) Show that a set of nonzero rational numbers is an ablian group under multiplication.

(i) When can a binary relation be called an equivalence relation?

(j) State whether the permutation $p=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 3 & 4 & 1\end{bmatrix}$ is odd or even permutation.

(k) A committee is formed from two men and two women. What is the probability that the committee will have no man?

(l) A problem in mechanics is given to three students $A$, $B$ and $C$ whose chance of solving it are $1/2$, $1/3$ and $1/4$ respectively. What is the probability that the problem will be solved?

(m) A biased six-sided dice has probabilities $\frac{p}{2}, p, p, p, p, 2p$ of getting 1, 2, 3, 4, 5, 6 respectively. Calculate $p$.

(n) The probability density function $\psi(x)$ of a continuous random variable $x$ is defined by the relation

$$\psi(x)=\begin{cases} A/x^3 & \text{for } 5\le x\le 10 \\ 0 & \text{elsewhere} \end{cases}$$

Evaluate $A$.

(o) If $P(A)=0.3$, $P(A\cup B)=0.6$, where $A$ and $B$ are independent then find $P(B)$.

(p) The mean and standard deviation of a binomial distribution are 10 and 2 respectively. Find the probability of success in each trial.

(q) A random variable $x$ has a probability density function $f(x)=e^{-x}$ in the interval $0<x<\infty$, and zero elsewhere. Find the value of probability that $x$ lies in the interval $1\le x\le 2$.

(r) Show that the equation $\frac{\partial^2 u}{\partial t^2}-c^2\frac{\partial^2 u}{\partial x^2}=0$ is a hyperbolic differential equation.

(s) Find the solution of the equation $\frac{\partial u}{\partial t}+3\frac{\partial u}{\partial x}=4x$ with $u(x,0)=6e^{-3t}$.

(t) Show that the function, $U(x,y)=f(x+iy)+g(x-iy)+x^3+y^3$ satisfies the partial differential equation $\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}=6(x+y)$ where $f$ and $g$ are arbitrary functions and $i=\sqrt{-1}$.

2. (a) Show that the set of four $2\times 2$ matrices given by 4+1

$$A=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, B=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, C=\begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix}, D=\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$$ forms a group of order four under multiplication and the group is abelian.

(b) Prove that the elements common to two intersecting subgroups $H_1$ and $H_2$ form a subgroup of the group $G$. 3

(c) Prove that a group where order is a prime number must be a cyclic group. 2

3. (a) Show that a non-empty subset $H$ of a group $G$ is a subgroup of $G$ if $a\in H$ and $b\in H$ implies $ab^{-1}\in H$. 3

(b) Show that the mapping $f:(Z,+)\to(2Z,+)$ such that $f(x)=2x,\ \forall x\in Z$ is an isomorphism from $Z$ to $2Z$, where $Z$ is the set of integers. 3

(c) A density function is defined by 2+2

$$f(x)=\begin{cases} e^{-x}, & x\ge 0 \\ 0, & x<0 \end{cases}.$$

(i) Determine the probability that the variate having this density will fall in the interval (1, 2).

(ii) Find the cumulative probability function $F(2)$.

4. (a) What are Dirichlet and Neumann boundary conditions? 2

(b) What are characteristic curves or characteristics of partial differential equations? 2

(c) An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of a scooter, car and a truck meeting an accident are 0.01, 0.03, 0.15 respectively. If one of the insured persons meets with an accident, find the probability that he is a scooter driver. 4

(d) Show that every subgroup of an abelian group is a normal subgroup. 2

5. (a) Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x \partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$ 5

(b) The Gaussian distribution with mean $\mu$ and standard deviation $\sigma$ is given by 3

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Find $\langle x \rangle$ and $\langle x^2 \rangle$.

(c) Show that the function $u=f\left(\frac{x}{y}\right)$ satisfies the equation $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$. 2

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